Int. J. Heat Mass Transfer. Vol. 31, No. 11, pp. 2227–2238, 1988
Printed in Great Britain

0017-9310/88 \$3.00+0.00

The effect of matrix longitudinal heat conduction on the temperature fields in the rotary heat exchanger

T. SKIEPKO†

Department of Heat Engineering, Technical University of Białystok, ul. Wiejska 45A, 15-351 Białystok, Poland

(*Received* 30 *July* 1987)

Abstract—In this paper two models describing transport phenomena in rotary heat exchangers are considered: one disregarding and the other including heat conduction in the matrix. Both models are described by the system of energy conservation equations which is solved by analytical methods. On the basis of these solutions the effect of the matrix longitudinal heat conduction on the temperature fields of gases and matrix is studied.

1. INTRODUCTION

APPLICATIONS of the rotary heat exchangers as thermal regenerators for steam boilers, gas turbine installations or ventilation and air conditioning systems are generally known. This is due to large and non-expensive heat transfer area per unit volume in the rotary regenerators (~ 400 m^2 m^{-3} for steam boiler regenerators and as many as ~ 2000 m^2 m^{-3} for gas turbine installations). Thus the counterflow rotary heat exchangers combine compactness with high performance.

Heat transport phenomena in the rotary exchangers have been modelled by systems of partial differential equations formulated with various simplifying assumptions. These models for steady-state operation can be classified into two categories:

(a) taking into consideration both convection and exchange terms [1–9] and thus conservation energy equations in non-dimensional form can be written as

$$\frac{\partial \vartheta_j}{\partial \varphi} = a_j(-\vartheta_j+\theta_j), \quad \frac{\partial \theta_j}{\partial z} = b_j(\vartheta_j-\theta_j), \quad j = 1,2; \tag{1}$$

(b) considering not only the terms mentioned above but also the term of matrix longitudinal heat conduction [10–14] so that the governing equations in dimensionless form are as follows:

$$\left.\begin{aligned} \frac{\partial \vartheta_j}{\partial \varphi} &= a_j(-\vartheta_j+\theta_j)+c_j\frac{\partial^2 \vartheta_j}{\partial z^2} \\ \frac{\partial \theta_j}{\partial z} &= b_j(\vartheta_j-\theta_j), \quad j = 1,2. \end{aligned}\right\} \tag{2}$$

For both categories of the models the balance equations are solved either analytically [1, 4, 6–9, 14] or numerically [2, 3, 5, 10–13]. These solutions are usually presented in the form of a relationship η–NTU_o when heat conduction is neglected [5] or η–NTU_o at $\lambda^* =$ const. taking heat conduction into consideration [11, 15]. In some papers [8, 9, 12–14] the solutions of the energy equations lead to the determination of temperature fields of the gases and matrix. The matrix thermal-conduction effect was evaluated by Mondt [10]. He found that the matrix thermal conduction causes a reduction of the temperature drops in the rotary heat exchanger.

The purpose of this paper is to determine the effect of the matrix longitudinal heat conduction on the temperature fields of the gases and matrix. The effect of the heat conduction is analysed by comparing the calculated temperature fields obtained from the solutions of equation systems (1) and (2). The solution of the system of equations (1) can be easily expressed by *Bs* and *Bes* functions which have been applied in the theory of heat exchangers by Łach [16]. The solution for the system of equations (2) was taken from ref. [14].

2. THE SOLUTION AT $\lambda_m = 0$

The coordinate system for these considerations is shown in Fig. 1. The energy conservation equations can be written in the form

$$\left.\begin{aligned} (1-\varepsilon)\rho_m c_m \omega \frac{\partial t_j}{\partial \phi} &= \alpha_j Y(T_j-t_j) \\ \varepsilon \rho_j c_{pj} v_j \frac{\partial T_j}{\partial \zeta} &= \alpha_j Y(t_j-T_j), \\ j &= 1,2 \end{aligned}\right\} \tag{3}$$

with boundary conditions

$$T_1(\phi,\zeta=0) = T'_1(\phi) \tag{4}$$

$$T_2(\phi,\zeta=0) = T'_2(\phi) \tag{5}$$

† Postal address: ul. Kalinowa 9 m.6, 15-809 Białystok, Poland.

NOMENCLATURE

a, b, c	coefficients
A_k, D_k	coefficients of the series for heating and cooling zones, respectively
c_p	gas specific heat at constant pressure
c_m	metal matrix specific heat
C_k	coefficients of the numerical quadrature for integral equations
d	distance between temperature fields
h	matrix height
r_1, s_1	real roots of the characteristic equations for heating and cooling zones, respectively
r_2, s_2	real parts of complex conjugate roots of the characteristic equations for heating and cooling zones, respectively
r_3, s_3	imaginary parts of complex conjugate roots of the characteristic equations for heating and cooling zones, respectively
t	matrix temperature
T	gas temperature
v	velocity of gas in matrix
Y	matrix heat transfer area per unit volume.

Greek symbols

α	heat transfer coefficient
β, γ	coefficients
ε	porosity
ζ, ϕ	coordinates, along the matrix in the direction of gas flow and rotation of the matrix, respectively
η	exchanger heat transfer effectiveness
λ_m	thermal conductivity of matrix
μ^2, ν^2	roots of the transcendental equations for heating and cooling zones, respectively
ρ, ρ_m	density of gas and metal matrix, respectively
ψ	zone angle
ω	rotational speed.

Subscripts

1	heating zone
2	cooling zone
k	serial number
m	matrix.

Superscript

$'$	at the inlet.

Dimensionless quantities

NTU	number of transfer units for gas, $\alpha Yh/(\varepsilon\rho c_p v)$
NTU_m	number of transfer units for metal matrix, $\alpha Y\psi/(1-\varepsilon)\rho_m c_m \omega]$
NTU_o	overall number of transfer units
Pe	Peclet number of metal matrix, $[\lambda_m\psi/(\rho_m c_m \omega h^2)]^{-1}$
z	longitudinal coordinate, ζ/h
ϑ	matrix temperature, $(t-T'_2)/(T'_1-T'_2)$
θ	gas temperature, $(T-T'_2)/(T'_1-T'_2)$
λ^*	conduction parameter (Bahnke and Howard [11])
φ	coordinate in direction of rotation, ϕ/ψ.

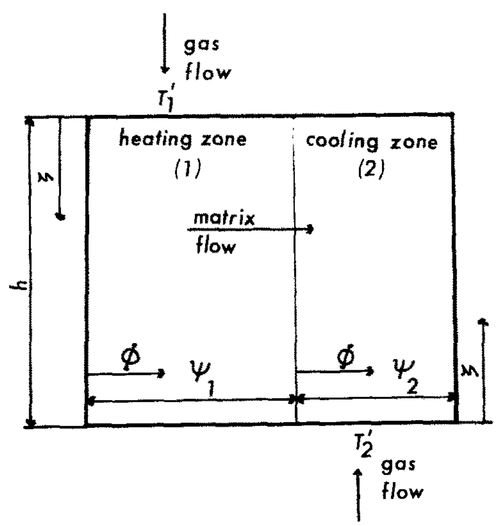

FIG. 1. Coordinate system and schematic representation of the problem under consideration for the rotary heat exchanger.

$$t_1(\phi = 0, \zeta) = t_2(\phi = \psi_2, h-\zeta) \tag{6}$$

$$t_1(\phi = \psi_1, \zeta) = t_2(\phi = 0, h-\zeta). \tag{7}$$

The system of equations (3) may be rewritten in dimensionless form by introducing new non-dimensional quantities mentioned in the nomenclature

$$\left.\begin{aligned} \frac{\partial \vartheta_j}{\partial \varphi} &= NTU_{mj}(-\vartheta_j + \theta_j) \\ \frac{\partial \theta_j}{\partial z} &= NTU_j(\vartheta_j - \theta_j), \quad j = 1, 2. \end{aligned}\right\} \tag{8}$$

The dimensionless boundary conditions are

$$\theta_j(\varphi, z = 0) = f_j(\varphi), \quad j = 1, 2 \tag{9}$$

$$\vartheta_1(\varphi = 0, z) = \vartheta_2(\varphi = 1, 1-z) \tag{10}$$

$$\vartheta_1(\varphi = 1, z) = \vartheta_2(\varphi = 0, 1-z). \tag{11}$$

The solution of the system of equations (8) expressed by Bs and Bes functions for a crossflow recuperator was presented by Łach [16]. His solution is used here for each gas–matrix zone. Assuming that

$$T'_1 = \text{const.} \quad \text{and} \quad T'_2 = \text{const.} \tag{12}$$

one has boundary condition (9) in the form

$$\theta_1(\varphi, z=0) = 1 \tag{13}$$

$$\theta_2(\varphi, z=0) = 0. \tag{14}$$

Substituting relations (13) and (14) into the solutions [16] and having made the necessary integrations one obtains:

for the temperature of the matrix in the heating zone

$$\begin{aligned}\vartheta_1(\varphi, z) &= \vartheta_1(0,z)\,\mathrm{e}^{-NTU_{m1}\varphi} \\ &\quad + \int_0^z \vartheta_1(0,\mu)\,\mathrm{e}^{-NTU_{m1}\varphi - NTU_1(z-\mu)} \\ &\quad \times Bes_1(NTU_{m1}NTU_1\varphi, z-\mu)\,\mathrm{d}\mu \\ &\quad + \mathrm{e}^{-NTU_{m1}\varphi - NTU_1 z} Bs_1(NTU_{m1}\varphi, NTU_1 z);\end{aligned} \tag{15}$$

for the temperature of the gas in the heating zone

$$\begin{aligned}\theta_1(\varphi, z) &= \mathrm{e}^{-NTU_{m1}\varphi - NTU_1 z}\, Bs_0(NTU_{m1}\varphi, NTU_1 z) \\ &\quad + NTU_1 \int_0^z \vartheta_1(0,\delta)\,\mathrm{e}^{-NTU_{m1}\varphi - NTU_1(z-\delta)} \\ &\quad \times Bes_0\,[NTU_{m1}NTU_1(z-\delta), \varphi]\,\mathrm{d}\delta;\end{aligned} \tag{16}$$

for the temperature of the matrix in the cooling zone

$$\begin{aligned}\vartheta_2(\varphi, z) &= \vartheta_2(0,z)\,\mathrm{e}^{-NTU_{m2}\varphi} \\ &\quad + \int_0^z \vartheta_2(0,\mu)\,\mathrm{e}^{-NTU_{m2}\varphi - NTU_2(z-\mu)} \\ &\quad \times Bes_1(NTU_{m2}NTU_2\varphi, z-\mu)\,\mathrm{d}\mu;\end{aligned} \tag{17}$$

for the temperature of the gas in the cooling zone

$$\begin{aligned}\theta_2(\varphi, z) &= NTU_2 \int_0^z \vartheta_2(0,\delta)\,\mathrm{e}^{-NTU_{m2}\varphi - NTU_2(z-\delta)} \\ &\quad \times Bes_0\,[NTU_{m2}NTU_2(z-\delta), \varphi]\,\mathrm{d}\delta.\end{aligned} \tag{18}$$

The unknown functions $\vartheta_1(0, z)$ and $\vartheta_2(0, z)$ in formulas (15)–(18) must be obtained on the basis of boundary conditions (10) and (11). Setting relationships (15) and (17) into equations (10) and (11) the following system of integral equations is obtained:

$$\left.\begin{aligned}&\vartheta_1(0,z)\,\mathrm{e}^{-NTU_{m1}} + \int_0^z \vartheta_1(0,\mu)\,\mathrm{e}^{-NTU_{m1} - NTU_1(z-\mu)} \\ &\times Bes_1(NTU_{m1}NTU_1, z-\mu)\,\mathrm{d}\mu - \vartheta_2(0, 1-z) \\ &\qquad = -\mathrm{e}^{-NTU_{m1} - NTU_1 z} Bs_1(NTU_{m1}, NTU_1 z) \\ &\vartheta_1(0,z) - \vartheta_2(0,1-z)\,\mathrm{e}^{-NTU_{m2}} \\ &\quad - \int_0^{1-z} \vartheta_2(0,\mu)\,\mathrm{e}^{-NTU_{m2} - NTU_2(1-z-\mu)} \\ &\quad \times Bes_1(NTU_{m2}NTU_2, 1-z-\mu)\,\mathrm{d}\mu = 0.\end{aligned}\right\} \tag{19}$$

To find the solution of the above problem (19) the method of successive approximations may be applied. However, this procedure is too complicated to be carried out analytically. System (19) can be easily solved in a numerical way [2] by using the collocation method which reduces the problem into a system of linear equations. The latter was used here (Appendix A).

3. THE SOLUTION AT $\lambda_m > 0$

In this case the energy conservation equations (coordinate system in Fig. 1) are as follows:

$$\left.\begin{aligned}&(1-\varepsilon)\rho_m c_m \omega \frac{\partial t_j}{\partial \phi} \\ &\qquad = \alpha_j Y(T_j - t_j) + (1-\varepsilon)\lambda_m \frac{\partial^2 t_j}{\partial \zeta^2} \\ &\varepsilon \rho_j c_{pj} v_j \frac{\partial T_j}{\partial \zeta} = \alpha_j Y(t_j - T_j), \quad j = 1, 2\end{aligned}\right\} \tag{20}$$

subject to boundary conditions

$$T_j(\phi, \zeta = 0) = T'_j, \quad j = 1, 2 \tag{21}$$

$$t_1(\phi = 0, \zeta) = t_2(\phi = \psi_2, h-\zeta) \tag{22}$$

$$t_1(\phi = \psi_1, \zeta) = t_2(\phi = 0, h-\zeta) \tag{23}$$

$$\partial t_j\,[\phi, (\zeta = 0 \text{ and } h)]/\partial\zeta = 0, \quad j = 1, 2. \tag{24}$$

By introducing dimensionless quantities defined in the nomenclature one can reduce the system of equations (20) to

$$\left.\begin{aligned}&\frac{\partial \vartheta_j}{\partial \varphi} = NTU_{mj}(-\vartheta_j + \theta_j) + Pe_j^{-1}\frac{\partial^2 \vartheta_j}{\partial z^2} \\ &\frac{\partial \vartheta_j}{\partial z} = NTU_j(\vartheta_j - \theta_j), \quad j = 1, 2.\end{aligned}\right\} \tag{25}$$

The corresponding dimensionless boundary conditions are

$$\theta_1(\varphi, z = 0) = 1 \tag{26}$$

$$\theta_2(\varphi, z = 0) = 0 \tag{27}$$

$$\vartheta_1(\varphi = 0, z) = \vartheta_2(\varphi = 1, 1-z) \tag{28}$$

$$\vartheta_1(\varphi = 1, z) = \vartheta_2(\varphi = 0, 1-z) \tag{29}$$

$$\partial\vartheta_j\,[\varphi, (z = 0 \text{ and } 1)]/\partial z = 0, \quad j = 1, 2. \tag{30}$$

The solution of the above problem is given in ref. [14] (Appendix B) in the form of a series:

for the gas temperature in the heating zone

$$\begin{aligned}\theta_1(\varphi, z) &= 1 - \sum_{k=0}^{\infty} A_k\,\mathrm{e}^{-\mu_k^2\varphi + r_{2,k}z} \\ &\quad \times [\mathrm{e}^{(r_{1,k} - r_{2,k})z} - \cos(r_{3,k}z) + \beta_k \sin(r_{3,k}z)];\end{aligned} \tag{31}$$

for the matrix temperature in the heating zone

$$\vartheta_1(\varphi, z) = 1 - \sum_{k=0}^{\infty} A_k \, e^{-\mu_k^2 \varphi + r_{2,k} z} \times \left[e^{(r_{1,k} - r_{2,k})z} \left(1 + \frac{r_{1,k}}{NTU_1}\right) + \left(\frac{\beta_k r_{3,k} - r_{2,k}}{NTU_1} - 1\right) \times \cos(r_{3,k} z) + \left(\beta_k + \frac{\beta_k r_{2,k} + r_{3,k}}{NTU_1}\right) \sin(r_{3,k} z)\right]. \quad (32)$$

Similar formulas for temperature fields in the cooling zone are presented in Appendix B.

4. THE EFFECT OF MATRIX HEAT CONDUCTION

This effect was determined by comparison of temperature fields calculated on the basis of the solutions presented in Sections 2 and 3. In Figs. 2–7 are presented the temperature fields in the rotary heat exchanger at various Peclet numbers, NTU_m and NTU values. The effect of longitudinal heat conduction was evaluated by calculating the distance between temperature fields. This distance d was described by formulas:

for the temperature fields of gases in the j-zone

$$d_j = \sqrt{\left(\int_0^1 \int_0^1 [\theta_j(\varphi, z)|_{Pe_j^{-1}=0} - \theta_j(\varphi, z)|_{Pe_j^{-1}>0}]^2 \, dz \, d\varphi\right)}; \quad (33)$$

for the temperature fields of the matrix

$$d_m = \sqrt{\left(\sum_{j=1}^{2} \int_0^1 \int_0^1 [\vartheta_j(\varphi, z)|_{Pe_j^{-1}=0} - \vartheta_j(\varphi, z)|_{Pe_j^{-1}>0}]^2 \, dz \, d\varphi\right)}. \quad (34)$$

The computations of distances d or d_m were performed on the basis of the solutions presented in Sections 1 and 2. Numerical results are presented in Figs. 8–11 at $NTU_m = 0.2$ and 1.0, respectively.

The analysis of temperature fields presented in Figs. 2–7 shows that the trends of temperature changes in the function of the coordinates are similar both when heat conduction is neglected ($Pe^{-1} = 0$) or taken into account ($Pe^{-1} > 0$). The effect of longitudinal heat conduction makes the temperature of the gas at the outlet of the heating zone higher and the cooling zone lower as compared with the temperature of gases when heat conduction is not taken into account. It is a characteristic for calculated temperature fields of gases with regard to heat conduction that:

(a) the temperature decreases in the direction of gas flow through the heating zone more rapidly than when heat conduction is not taken into account, next the trend diminishes, the graphs of the temperature fields at $Pe^{-1} > 0$ and $Pe^{-1} = 0$ intersect and at the outlet the gas temperature is higher at $Pe^{-1} > 0$ than at $Pe^{-1} = 0$;

(b) the temperature increases in the direction of the

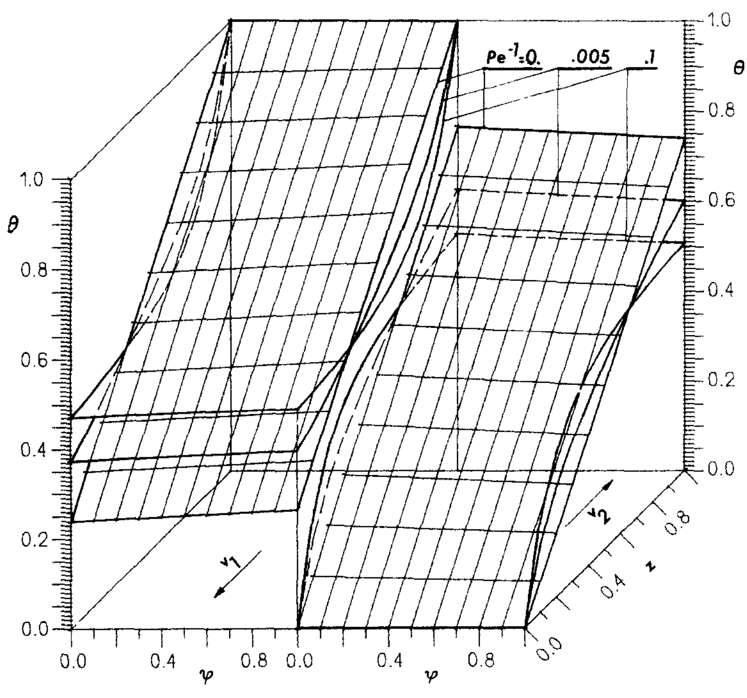

FIG. 2. Effect of matrix longitudinal heat conduction on temperature distributions of gases in rotary heat exchanger at constants: $NTU_1 = NTU_2 = 6$ and $NTU_{m1} = NTU_{m2} = 0.2$.

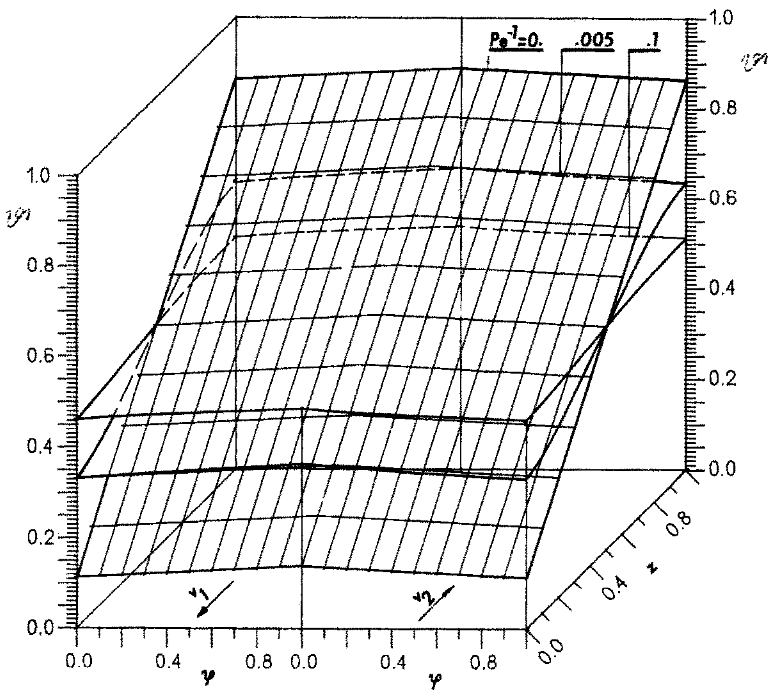

FIG. 3. Effect of matrix longitudinal heat conduction on temperature distributions of matrix in rotary heat exchanger at constants: $NTU_1 = NTU_2 = 6$ and $NTU_{m1} = NTU_{m2} = 0.2$.

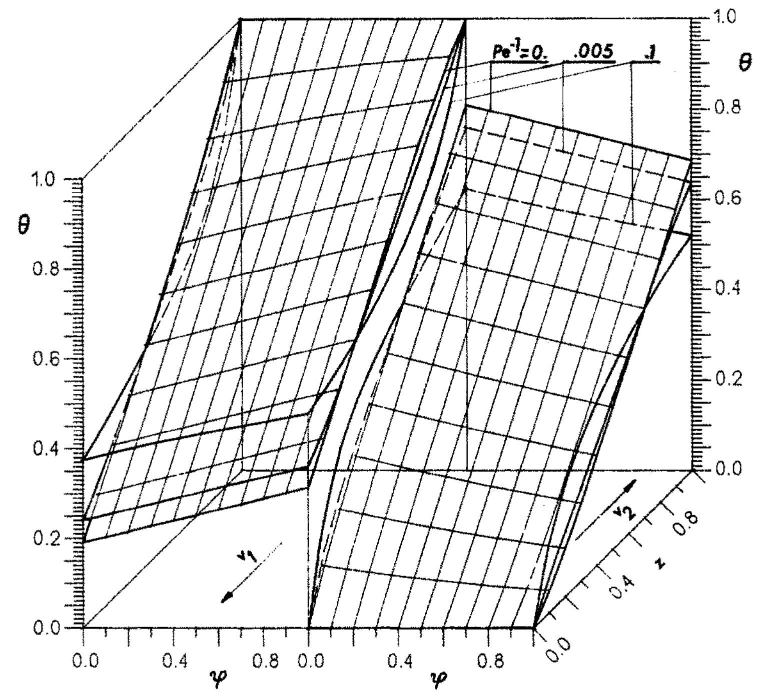

FIG. 4. Effect of matrix longitudinal heat conduction on temperature distributions of gases in rotary heat exchanger at constants: $NTU_1 = NTU_2 = 6$ and $NTU_{m1} = NTU_{m2} = 1$.

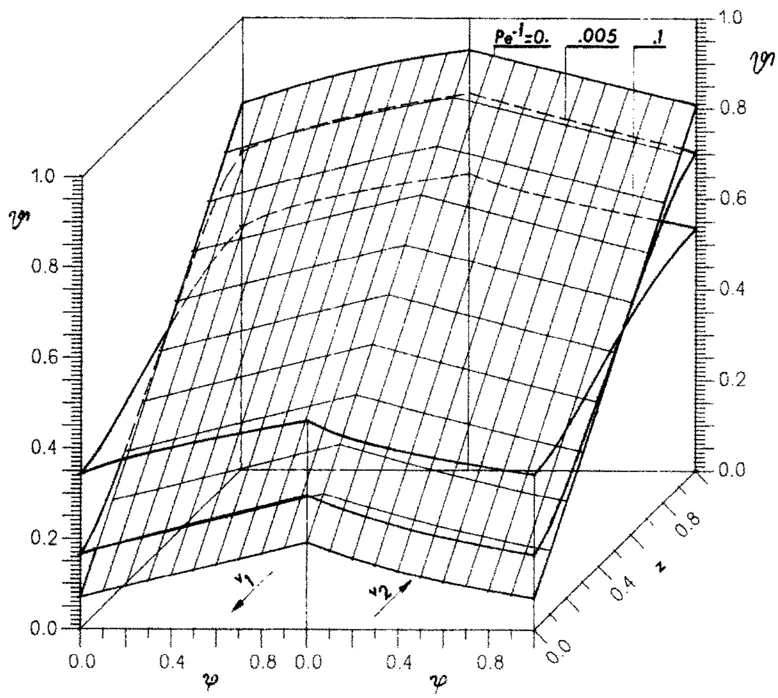

Fig. 5. Effect of matrix longitudinal heat conduction on temperature distributions of matrix in rotary heat exchanger at constants: $NTU_1 = NTU_2 = 6$ and $NTU_{m1} = NTU_{m2} = 1$.

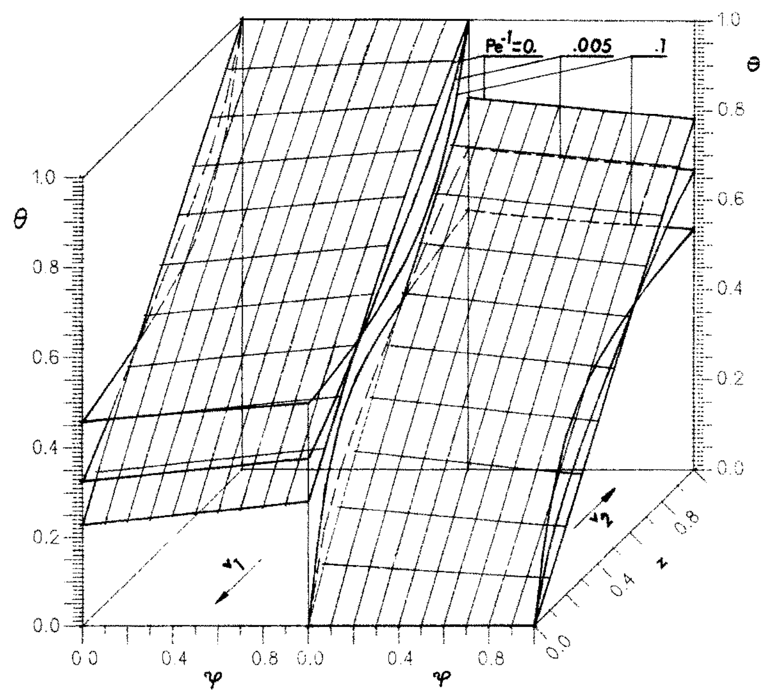

Fig. 6. Effect of matrix longitudinal heat conduction on temperature distributions of gases in rotary heat exchanger at constants: $NTU_1 = 6$ and $NTU_2 = 8$, $NTU_{m1} = 0.4$, $NTU_{m2} = 0.5$.

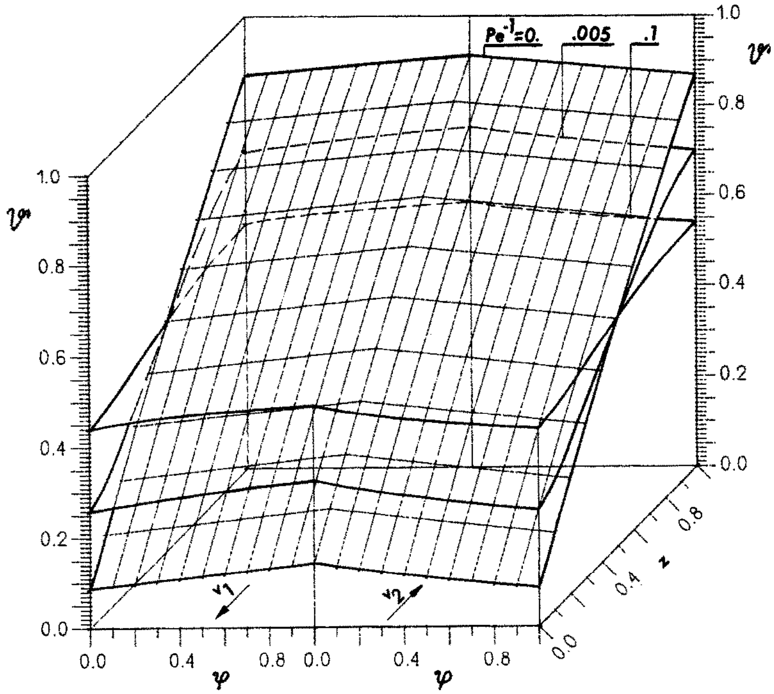

FIG. 7. Effect of matrix longitudinal heat conduction on temperature distributions of the matrix in rotary heat exchanger at constants: $NTU_1 = 6$, $NTU_2 = 8$, $NTU_{m1} = 0.4$, $NTU_{m2} = 0.5$.

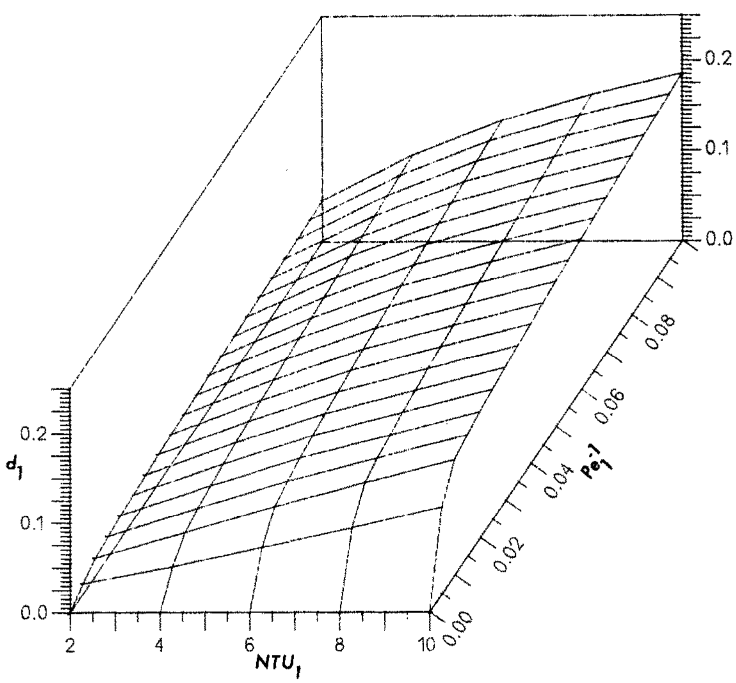

FIG. 8. Effect of NTU_1 and Pe_1 on the distance d_1 (formula (33)) between temperature fields of gases in the heating zone at constants: $NTU_1 = NTU_2$, $Pe_1 = Pe_2$, $NTU_{m1} = NTU_{m2} = 0.2$ (in the cooling zone the effect is identical).

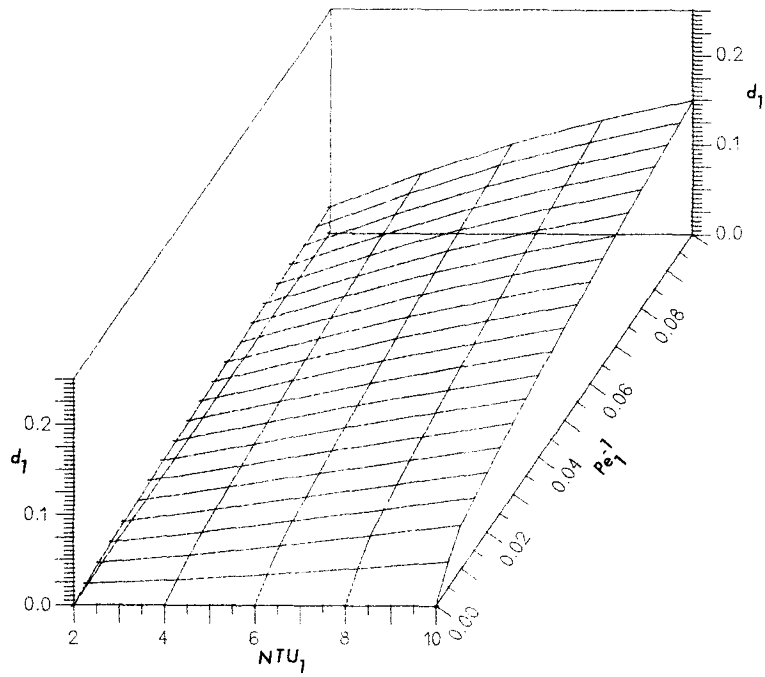

FIG. 9. Effect of NTU_1 and Pe_1 on the distance d_1 (formula (33)) between temperature fields of gases in the heating zone at constants: $NTU_1 = NTU_2$, $Pe_1 = Pe_2$ and $NTU_{m1} = NTU_{m2} = 1$ (in the cooling zone the effect is identical).

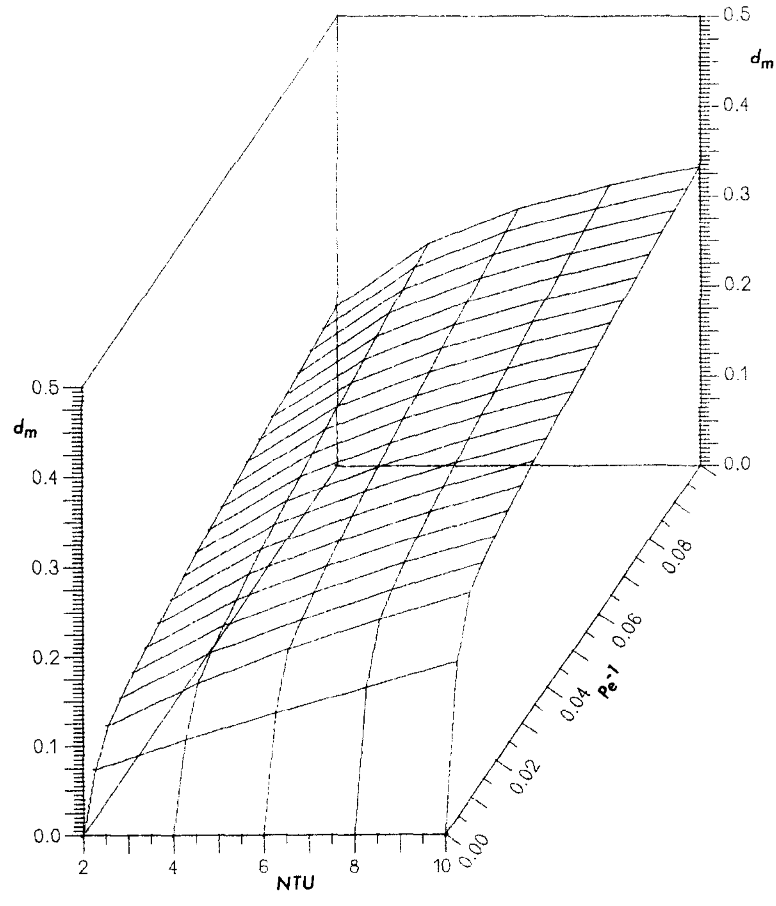

FIG. 10. Effect of $NTU_1 = NTU_2 = NTU$ and $Pe_1 = Pe_2 = Pe$ on the distance d_m (formula (34)) between temperature fields of matrix in rotary heat exchanger at constants: $NTU_{m1} = NTU_{m2} = 0.2$.

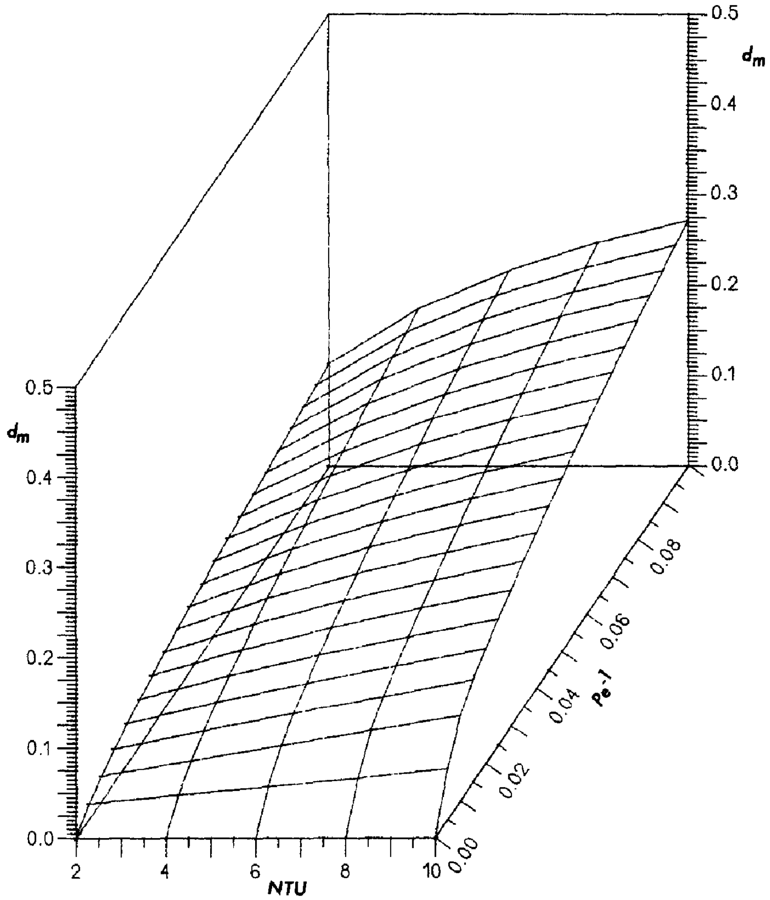

FIG. 11. Effect of $NTU_1 = NTU_2 = NTU$ and $Pe_1 = Pe_2 = Pe$ on the distance d_m (formula (34)) between temperature fields of matrix in rotary heat exchanger at constants: $NTU_{m1} = NTU_{m2} = 1$.

gas flow through the cooling zone more rapidly when $Pr^{-1} > 0$ than when $Pe^{-1} = 0$, next the trend decreases, the charts of temperature fields at $Pe^{-1} > 0$ and $Pe^{-1} = 0$ intersect and at the outlet of the cooling zone the gas temperature is lower at $Pe^{-1} > 0$ than $Pe^{-1} = 0$.

The temperature of the matrix changes in the direction of the gas flow as follows:

(a) at the inlet of the heating zone it is lower while it is higher at the inlet of the cooling zone at $Pe^{-1} > 0$ in comparison to $Pe^{-1} = 0$;

(b) at $Pe^{-1} > 0$ the temperature of the matrix changes by a lesser degree than at $Pe^{-1} = 0$, the charts of the temperature fields at $Pe^{-1} > 0$ and $Pe^{-1} = 0$ intersect;

(c) hence at the outlet of the zones the temperature of the matrix is higher in the heating zone and lower in the other zone at $Pe^{-1} > 0$ in comparison to $Pe^{-1} = 0$.

Finally, the numerical experiments presented here show that the effect of heat conduction in the matrix is greater on the matrix temperature field than on the gas temperature.

In Figs. 8–11 the influence of the heat conduction is shown on distances d and d_m obtained from formulas (33) and (34). As shown the distance d between temperature fields calculated at $Pe^{-1} > 0$ and $Pe^{-1} = 0$ changes as follows:

(a) with an increase of Pe^{-1} values the distance d also increases at NTU_m = const. and NTU = const.;

(b) as NTU increases the distance d also increases at NTU_m = const. and Pe^{-1} = const.;

(c) as NTU_m increases the distance d decreases at NTU = const. and Pe^{-1} = const.

Hence one may conclude that the effect of longitudinal heat conduction in the matrix is essential at small NTU_m values. Moreover, the effect becomes greater with an increase of Pe^{-1} and NTU values.

5. CONCLUDING REMARKS

On the basis of the analytical solutions of model equations the effect of the longitudinal heat conduction in the matrix on temperature fields of gases and matrix of the rotary heat exchangers was studied at the ranges of the dimensionless parameters in each gas matrix zone as follows:

$$0.2 \leqslant NTU_m \leqslant 1$$

$$1 \leqslant NTU \leqslant 10$$

$$0.005 \leqslant Pe^{-1} \leqslant 0.1.$$

The numerical experiments reported in this paper have shown that heat conduction may essentially affect the temperature fields in rotary heat exchangers. The influence is greater for the matrix than for the gases and particularly evident at small NTU_m values.

REFERENCES

1. W. Nusselt, Die Theorie des Winderhitzers, *Z. Ver. Dt. Ing.* **71,** 85–91 (1927).
2. C. E. Iliffe, Thermal analysis of the contra-flow regenerative heat exchanger, *Proc. Instn Mech. Engrs* **159,** 363–371 (1948).
3. O. A. Saunders and S. Smoleniec, Heat regenerators, *Proc. 7th Int. Congress for Appl. Mech.*, Vol. 3, pp. 91–105 (1948).
4. J. E. Coppage and A. L. London, The periodic-flow regenerator—a summary of design theory, *Trans. ASME* **75,** 779–787 (1953).
5. T. J. Lambertson, Performance factors of a periodic-flow heat exchanger, *Trans. ASME* **80,** 586–592 (1958).
6. V. K. Migaj, Regenerative rotary air preheaters (in Russian), *Energija* (1971).
7. J. Madejski, *Theory of Heat Transfer* (in Polish). PWN, Warszawa-Poznań (1963).
8. Neaga, Thermal calculation of regenerative air preheaters, *Revue Roum. Sci. Tech.-Electrotech. Energ.* **19,** 193–206 (1974).
9. F. E. Romie, Periodic thermal storage: the regenerator, *Trans. ASME, J. Heat Transfer* **101,** 726–731 (1979).
10. J. R. Mondt, Vehicular gas turbine periodic-flow heat exchanger solid and fluid temperature distributions, *Trans. ASME, J. Engng Pwr* **86,** 121–126 (1964).
11. G. D. Bahnke and C. P. Howard, The effect of longitudinal heat conduction on periodic-flow heat exchanger performance, *Trans ASME, J. Engng Pwr* **86,** 105–120 (1964).
12. K. Brodowicz, *Theory of Heat and Mass Exchangers* (in Polish). PWN, Warszawa (1982).
13. Chung-Hsiung Li, A numerical finite difference method for performance evaluation of a periodic-flow heat exchanger, *Trans. ASME, J. Heat Transfer* **105,** 611–617 (1983).
14. T. Skiepko, The solution of heat transport equations for a rotary regenerator (in Polish), *Archwm Termodynamiki* **8**(1), 2 (1987).
15. W. M. Kays and A. L. London, *Compact Heat Exchangers*, 2nd Edn. McGraw-Hill, New York (1964).
16. J. Łach, The exact solution of the Nusselt's model of the cross-flow recuperator, *Int. J. Heat Mass Transfer* **26,** 1597–1601 (1983).

APPENDIX A

(1) Necessary integrals for rewriting the solutions [16] in the form of equations (15)–(18) are

$$NTU_{mj}\int_0^{\varphi} e^{-NTU_{mj}(\varphi-\delta)-NTU_j z}\, Bes_0\,[NTU_{mj}\, NTU_j(\varphi-\delta), z]\,d\delta$$
$$= e^{-NTU_{mj}\varphi-NTU_j z}\, Bes_1(NTU_{mj}\,\varphi, NTU_j z) \quad (A1)$$

$$\int_0^{\varphi} e^{-NTU_{mj}(\varphi-\mu)-NTU_j z}\, Bes_1\,(NTU_{mj}\, NTU_j\, z, \varphi-\mu)\,d\mu$$
$$= e^{-NTU_j z}\,[e^{-NTU_{mj}\varphi}\, Bs_0(NTU_{mj}\varphi, NTU_j z)-1]. \quad (A2)$$

(2) The numerical form of the system of integral equations (19). The algebraic elements of equations (15)–(18) are denoted as follows:

$$K_1(z,\mu) = e^{-NTU_{m1}-NTU_1(z-\mu)}\, Bes_1\,(NTU_{m1}\, NTU_1, z-\mu) \quad (A3)$$

$$P(z) = -e^{-NTU_{m1}-NTU_1 z}\, Bs_1\,(NTU_{m1}, NTU_1 z) \quad (A4)$$

$$K_2(z,\mu) = e^{-NTU_{m2}-NTU_2(1-z-\mu)}\, Bes_1\,(NTU_{m2} NTU_2, 1-z-\mu). \quad (A5)$$

The trapezoid rule coefficients for the numerical presentation of the system of integral equations (19) are

$$\left.\begin{array}{ll} C_k = \Delta z, & k = 2,\ldots,i-1 \\ C_1 = C_i = \Delta z/2, & i = 1,\ldots,n. \end{array}\right\} \quad (A6)$$

By collocating the system of integral equations (19) at each $i = 1,\ldots,n$ collocation point one obtains an equivalent system of linear equations in the form

$$\left.\begin{array}{l} e^{-NTU_{m1}}\,\vartheta_1(0,z_1)-\vartheta_2(0,z_n) = \\ P(z_1)\sum\limits_{k=1}^{i-1} C_k\,\vartheta_1(0,z_k)K_1(z_i,z_k)+[e^{-NTU_{m1}}+C_iK_1(z_i,z_i)] \\ \quad\times\vartheta_1(0,z_i)-\vartheta_2(0,z_{n-i+1}) = P(z_i), \quad i=2,\ldots,n \\ \vartheta_1(0,z_i)-\sum\limits_{k=1}^{n-i} C_k\vartheta_2(0,z_k)K_2(z_i,z_k) \\ \quad -[e^{-NTU_{m2}}+C_iK_2(z_i,z_{n-i+1})]\,\vartheta_2(0,z_{n-i+1}) = 0, \\ \qquad\qquad i = 1,\ldots,n-1 \\ \vartheta_1(0,z_n)-e^{-NTU_{m2}}\vartheta_2(0,z_1) = 0. \end{array}\right\} \quad (A7)$$

After solving the above system of equations (A7) one obtains $\vartheta_1(0, z_i)$ and $\vartheta_2(0, z_i)$ values at each point ($i = 1,\ldots,n$) of the collocation.

APPENDIX B. THE SOLUTION OF THE ENERGY CONSERVATION EQUATION AT $\lambda_m > 0$

(1) The heating zone

By introducing new functions for the zone

$$\theta_1^*(\varphi,z) = 1-\theta_1(\varphi,z) \quad (B1)$$

$$\vartheta_1^*(\varphi,z) = 1-\vartheta_1(\varphi,z) \quad (B2)$$

system (25) can be written in the form

$$\left.\begin{array}{l} \dfrac{\partial\vartheta_1^*}{\partial\varphi} = NTU_{m1}(-\vartheta_1^*+\theta_1^*)+Pe_1^{-1}\dfrac{\partial^2\vartheta_1^*}{\partial z^2} \\ \dfrac{\partial\theta_1^*}{\partial z} = NTU_1(\vartheta_1^*-\theta_1^*). \end{array}\right\} \quad (B3)$$

Now, boundary conditions (26)–(30) may be given as follows:

$$\theta_1^*(\varphi, z=0) = 0 \quad (B4)$$

$$\theta_2^*(\varphi, z=0) = 0 \quad (B5)$$

$$1-\vartheta_1^*(\varphi=0,z) = \vartheta_2^*(\varphi=1,1-z) \quad (B6)$$

$$1-\vartheta_1^*(\varphi=1,z) = \vartheta_2^*(\varphi=0,1-z) \quad (B7)$$

$$\partial\vartheta_1^*[\varphi,(z=0 \text{ and } 1)]/\partial z = 0 \quad (B8)$$

$$\partial\vartheta_2^*[\varphi,(z=0 \text{ and } 1)]/\partial z = 0. \quad (B9)$$

From equations (B3) one can determine ϑ_1^*

$$\vartheta_1^* = \theta_1^* + \frac{1}{NTU_1}\frac{\partial\theta_1^*}{\partial z} \quad (B10)$$

and relationship (B10) is substituted into equation (B3). As a result one has

$$\frac{\partial \theta_1^*}{\partial \varphi}+\frac{1}{NTU_1}\frac{\partial^2\theta_1^*}{\partial z\,\partial\varphi} = Pe_1^{-1}\left(\frac{\partial^2\theta_1^*}{\partial z^2}+\frac{1}{NTU_1}\frac{\partial^3\theta_1^*}{\partial z^3}\right)-\frac{NTU_{m1}}{NTU_1}\frac{\partial\theta_1^*}{\partial z}. \quad (B11)$$

This equation will be solved by the method of separating variables. To do this the solution of equation (B11) is constructed in the form of the product

$$\theta_1^*(\varphi,z) = F(\varphi)Z(z). \quad (B12)$$

Substituting equation (B12) into equation (B11) one obtains, after separating variables, the following ordinary differential equations:

$$\frac{dF}{d\varphi}+\mu^2F = 0 \quad (B13)$$

$$\frac{Pe_1^{-1}}{NTU_1}\frac{d^3Z}{dz^3}+Pe_1^{-1}\frac{d^2Z}{dz^2}-\left(\frac{NTU_{m1}}{NTU_1}-\frac{\mu^2}{NTU_1}\right)\frac{dZ}{dz}+\mu^2 Z = 0. \quad (B14)$$

The general solution of equation (B13) has the form

$$F(\varphi) = \text{const.}\exp(-\mu^2\varphi) \quad (B15)$$

while

$$\frac{Pe_1^{-1}}{NTU_1}x^3+Pe_1^{-1}x^2-\left(\frac{NTU_{m1}}{NTU_1}-\frac{\mu^2}{NTU_1}\right)x+\mu^2 = 0 \quad (B16)$$

is a characteristic equation for a linear homogeneous differential equation (B14). The μ^2-values are chosen in such a way that equation (B16] has one real root, r_1 and two complex conjugate roots, $r_2' = r_2+ir_3$ and $r_3' = r_2-ir_3$, In this way boundary conditions (B4) and (B8) can be satisfied simultaneously. The real solution of equation (B14) has the form

$$Z(z) = A\,e^{r_1 z}+e^{r_2 z}[B\cos(r_3 z)+C\sin(r_3 z)]. \quad (B17)$$

Taking into account expressions (B10) and (B12) one can rewrite boundary conditions (B4) and (B8) in the form

$$Z|_{z=0} = 0 \quad (B18)$$

$$\frac{d}{dz}\left[Z+\frac{1}{NTU_1}\frac{dZ}{dz}\right]_{z=0} = 0 \quad (B19)$$

$$\frac{d}{dz}\left[Z+\frac{1}{NTU_1}\frac{dZ}{dz}\right]_{z=1} = 0. \quad (B20)$$

Taking into consideration solution (B17), on the basis of conditions (B18)–(B20), one obtains a homogeneous system of three linear equations

$$\begin{bmatrix} 1 & 1 & 0 \\ a & b & c \\ a\,e^{r_1} & e^{r_2}[b\cos(r_3)-c\sin(r_3)] & e^{r_2}[b\sin(r_3)+c\cos(r_3)] \end{bmatrix} \times \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (B21)$$

where

$$a = r_1\left(1+\frac{r_1}{NTU_1}\right) \quad (B22)$$

$$b = r_2+\frac{r_2^2-r_3^2}{NTU_1} \quad (B23)$$

$$c = r_3+\frac{2r_2r_3}{NTU_1}. \quad (B24)$$

The linear homogeneous system (B21) has non-zero solutions if its determinant is equal to zero. Hence one obtains the following transcendental equation:

$$ac\,e^{r_1-r_2}+(b^2-ab+c^2)\sin(r_3)-ac\cos(r_3) = 0 \quad (B25)$$

on the eigenvalues μ^2. Equation (B25) has successively increasing non-negative roots ($k = 0, 1, \ldots, \infty$). Now equation (B21) can be rewritten in the form

$$\begin{bmatrix} 1 & 0 \\ b_k & c_k \end{bmatrix}\begin{bmatrix} B_k \\ C_k \end{bmatrix} = \begin{bmatrix} -A_k \\ -A_k a_k \end{bmatrix} \quad (B26)$$

Hence one obtains

$$B_k = -A_k,\quad C_k = A_k\beta_k,\quad \beta_k = (b_k-a_k)/c_k,\quad k = 0,1,\ldots \quad (B27)$$

Substituting equations (B27) into equation (B17) one obtains the eigenfunctions Z_k as follows:

$$Z_k(z) = A_k\,e^{r_{2,k}}[e^{(r_{1,k}-r_{2,k})z}-\cos(r_{3,k}z)+\beta_k\sin(r_{3,k}z)],\quad k = 0,1,2,\ldots \quad (B28)$$

After substituting equation (B28) into equation (B12) and also taking into account equations (B1), (B2), (B10) and (B15) one finally obtains the solutions in the form of equations (31) and (32).

(2) The cooling zone

By making use of a similar procedure as above one obtains solutions of the energy conservation equations for this zone in the forms:

(a) gas temperature

$$\theta_2(\varphi,z) = \sum_{k=0}^{\infty} D_k\,e^{-v_k^2\varphi+s_{2,k}z} \times [e^{(s_{1,k}-s_{2,k})z}-\cos(s_{3,k}z)+\gamma_k\sin(s_{3,k}z)]; \quad (B29)$$

(b) matrix temperature

$$\vartheta_2(\varphi,z) = \sum_{k=0}^{\infty} D_k\,e^{-v_k^2\varphi+s_{2,k}z}\,[e^{(s_{1,k}-s_{2,k})z} \times\left(1+\frac{s_{1,k}}{NTU_2}\right)+\left(\frac{\gamma_k s_{3,k}-s_{2,k}}{NTU_2}-1\right)\cos(s_{3,k}z) +\left(\gamma_k+\frac{\gamma_k s_{2,k}+s_{3,k}}{NTU_2}\right)\sin(s_{3,k}z)]. \quad (B30)$$

Coefficients A_k and D_k are determined on the basis of boundary conditions (B6) and (B7) by the collocation method.

EFFET DE LA CONDUCTION THERMIQUE LONGITUDINALE, DANS LA MATRICE, SUR LE CHAMP DE TEMPERATURE DANS UN ECHANGEUR TOURNANT

Résumé—On considère deux modèles pour décrire les phénomènes de transfert dans des échangeurs de chaleur rotatifs : l'un néglige et l'autre prend en compte la conduction de chaleur dans la matrice. Les deux modèles utilisent les équations de conservation d'énergie qui sont résolues par des méthodes analytiques. A partir de ces solutions est étudié l'effet de la conduction longitudinale de matrice sur les champs de température dans le gaz et la matrice.

DIE AUSWIRKUNG DER LÄNGSWÄRMELEITUNG AUF DAS TEMPERATURFELD IN EINEM ROTIERENDEN WÄRMETAUSCHER

Zusammenfassung—In dieser Arbeit werden zwei Modellvarianten untersucht, die Transportvorgänge in rotierenden Wärmetauschern beschreiben : Ein Modell berücksichtigt die Wärmeleitung, das andere nicht. Beide Modelle werden durch das System der Energieerhaltungsgleichungen beschrieben, die durch analytische Methoden gelöst werden. Auf der Basis dieser Lösungen wird die Auswirkung der Längswärmeleitung in der Matrix auf die Temperaturfelder in der Matrix und in den Gasen untersucht.

ВЛИЯНИЕ ПРОДОЛЬНОЙ ТЕПЛОПРОВОДНОСТИ МАТРИЦЫ НА ТЕМПЕРАТУРНЫЕ ПОЛЯ В РОТОРНЫХ ТЕПЛООБМЕННИКАХ

Аннотация—Рассмотрены две модели, описывающие явления переноса в роторных теплообменниках: в одной из них игнорируется, а во второй учитывается теплопроводность матрицы. Обе модели содержат систему уравнений сохранения энергии, которые решаются аналитическими методами. На основе этих решений исследуется влияние продольной теплопроводности матрицы на распределение температуры в ней и газах.